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RAPID STRUCTURAL DESIGN CHANGE EVALUATION WITH AN EXPERIMENT BASED FEM

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The work in this paper proposes a dynamic structural design model that can be developed in a rapid fashion. The approach endeavours to produce a simplified FEM developed in conjunction with an experimental modal database. The FEM is formulated directly from the geometry and connectivity used in an experimental modal test using beam/frame elements. The model sacrifices fine detail for a rapid development time. The FEM is updated at the element level so the dynamic response replicates the experimental results closely. The physical attributes of the model are retained, making it well suited to evaluate the effect of potential design changes. The capabilities are evaluated in a series of computational and laboratory tests. First, a study is performed with a simulated cantilever beam with a variable mass and stiffness distribution. The modal characteristics serve as the updating target with random noise added to simulate experimental uncertainty. A uniformly distributed FEM is developed and updated. The results show excellent results, all natural frequencies are within 0·001% with MAC values above 0·99. Next, the method is applied to predict the dynamic changes of a hardware portal frame structure for a radical design change. Natural frequency predictions from the original FEM differ by as much as almost 18% with reasonable MAC values. The results predicted from the updated model produce excellent results when compared to the actual hardware changes, the first five modal natural frequency difference is around 5% and the corresponding mode shapes producing MAC values above 0·98.

1. INTRODUCTION

A need often arises in engineering practice to predict the structural dynamic behavior of an existing mechanical system for a potential design change. The proposed change may be necessary to resolve an identified dynamics and/or vibration problem and/or to improve performance. In many actual situations, a rapid response is more imperative than an in-depth analysis. The details can be sacrificed for a rapid indication of the correct design direction to be pursued. Finite element analysis (FEA) is well suited to this design evaluation task. However, the model development and verification process can be quite time and manpower intensive. Hence, the FEM development time is often prohibitive under these constraints. An alternative is to use the structural dynamic modification (SDM) approach in conjunction with an experimental modal database. However, SDM changes are typically limited (i.e., point to point spring and point masses). Furthermore, a SDM is not easily translated into an actual hardware change to the system (i.e., section area changes, material changes, weldment).

The work, presented in this paper, examines a method focused on this time pressured design scenario. The method uses an effective FEM developed by an element level updating

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process using an experimental modal database. The FEM is formulated directly from the experimental modal test geometry and connectivity. The FEM is restricted to beam/frame elements with a somewhat coarse discretization mesh. Obviously, the coarse character of the effective model will not have the capabilities of a fully developed FEM and will be insufficient for certain analysis (i.e., stress analysis). However, the effective FEM strives to obtain the global dynamic characteristics (i.e., first 10 natural frequencies and mode shapes). The effective FEM has the characteristics: (1) rapid development time; (2) the original model's physical significance is retained; (3) similar dynamic behavior as the experimental modal analysis results; (4) the model allows hardware parameters to be easily changed (i.e., dimensions and material properties) and items added (i.e., a strut). These characteristics make the proposed method appear well suited to perform rapid design change evaluations for existing hardware.

2. EFFECTIVE FEM METHOD DEVELOPMENT

The proposed method is based on a coarse mesh FEM using beam/frame elements. Any redesign requires that the initial FEM be sufficiently accurate to perform the analysis. In reality, this assumption is often not valid and the FEM usually needs to be "fine tuned" or "updated" to achieve the required level of accuracy. Two major questions arise concerning this approach: (1) how accurate and effective can a FEM be based on beam/frame elements and a coarse mesh; (2) how can the FEM be updated in a fashion which will allow effective redesign? The following sections address these concerns.

2.1. MODELLING WITH FRAME ELEMENTS

FEM modelling requires a variety of decisions such as the continuum discretization, element selection and boundary conditions. The model development depends on the desired results and the analysts' engineering judgment. Obviously, the more detailed the analysis (i.e., localized stresses) the more complicated the modelling (i.e., element types, number of elements). However, a number of studies have demonstrated that it is possible to accurately obtain low order global dynamic characteristics (i.e., natural frequencies and mode shapes) and forced vibration response using a coarse mesh and frame elements. For example, dynamic results for an automotive engine cradle obtained from a plate element FEA model (a fine mesh with over 16 000 DOF) compared very favorably to a beam element FEM with less than 500 DOF [1]. For eigenvalue analysis purposes it was demonstrated that a uni-body automotive structure can be modelled with a relatively few number of beam elements [2]. It was also shown in reference [3] that the dynamic response of an automobile structure is primarily influenced by its major load carrying members, with the sheet metal sections being far less important. Hence, it was concluded that beam elements lend themselves well toward modal studies of uni-body automotive structures. It was also suggested that beam element FEMs of uni-body vehicle structures can be made generic, in the sense that different designs can be modelled by merely changing beam dimensions [3]. For example, the passenger compartment can readily be "stretched", the trunk "shortened", or the roof "widened". In references [4,5] a complete automobile, including suspension, was modelled with approximately 200 frame elements. The model produced good correlation with experimental modal analysis results of the corresponding automobile structure. Forced vibration analysis using road profiles as the wheel inputs computed using the model correlated well with actual vehicle recorded responses. Additionally, the main support structure of a high speed precision machine tool was analyzed with as few as 10 elements [7] yielding excellent correlation to actual experimental responses measured during a work sequence. Hence in certain situations, it is possible to obtain accurate modal characteristics and forced response predictions for complicated structures with relatively low numbers of frame elements [8].

2.2. FEM UPDATING

To improve the ability of a FEM to respond in a similar fashion as the mechanical hardware requires some form of correlation with experimental data. Due to the critical design reliance on these FEMs, a significant amount of research has recently been devoted to developing updating methods [9]. Most approaches use data from an experimental modal analysis as the target and adjust the FEM's parameters until the natural frequencies and mode shapes agree. Only after the confidence of the FEM has been ensured, can it be applied to investigate effectively the dynamic response of a structure.

A variety of FEM updating methods have been proposed and can be separated into two broad categories: (1) direct updating using experimental modal test results via closed form optimization based procedures and (2) updating via the eigenvalue and eigenvector sensitivity with respect to the system's parameters. The basic idea relies on orthogonality properties, between the mathematical model (mass/stiffness matrix) and measured quantities (mode shapes/natural frequencies). Several closed form relationships [10–12] have been proposed. The techniques use the orthogonality characteristics while minimizing the changes of the weighted mass/stiffness matrices. To implement the closed form updating, either matrix reduction [13, 14] or mode shape expansion [15, 16] is required. Matrix reduction suffers from the inability to recover the full size matrices after updating. Whereas, mode shape expansion usually is computationally intensive and is suitable only when the initial FEM is relatively accurate. The updated results reveal that original element connectivity is destroyed and hence the physical significance of the resulting FEM mass and stiffness matrices (i.e., negative values at some specific DOFs and entries appearing which were previously non-existent). Further application of the updated FEM for design change evaluation is precluded. Procedures have been proposed [17] that retain element connectivity but the positive definite characteristic of the updated matrices is not assured.

Alternative updating approaches use the sensitivities of the dynamic system's eigenvalues and eigenvectors with respect to the parameters [18–20]. The basic eigendata sensitivity formulation appears straightforward [21], however practical application can be tedious and difficult [22]. There are two broad categories, underdetermined and overdetermined methods. The underdetermined process has fewer constraints than design variables and the cost function is optimally minimized subject to constraints. The overdetermined process uses a least squares approach to minimize the constraint error.

The use of experimental modal analysis results to update a FEM has been widely explored recently. Unfortunately, no single method has become widely accepted. The reported successful applications have been developed and applied to specific types of problems. All of the present updating methods have at least one or more of the following problems: (1) loss of physical significance in the updated mass and stiffness matrices with respect to the original model; (2) inaccuracy of the results; (3) sensitivity to uncertainties in the experimental modal data; (4) numerical difficulties which affect convergence to a final solution.

Most updating methods strive for good correlation between the computational and experimental results at the expense of the model's physical significance. From a design point of view the physical significance of the model must be retained.

2.3. ELEMENTAL LEVEL FEM UPDATING

The effective FEM is based on a sensitivity scheme and uses the concept of updating at the element level. The updating variables are contained in a vector, $\{\mathbf{P}\}$, and can be tailored to the problem. The updating variables may represent a multiplicative constant for each element or may be more specific such as each element's cross-sectional area (A) or area moment of inertia (I).

In this work, $[\mathbf{m}]^i_{FEA}$ and $[\mathbf{k}]^i_{FEA}$ represent the elemental mass and stiffness matrices for the ith element and $\{\mathbf{P}_i\}$ the updating variables. A nomenclature list appears in the Appendix. The updating is accomplished by establishing the sensitivity of the system's eigendata with respect to each individual element's physical properties (i.e., density, cross-sectional area, and moment of inertia). A classical quadratic optimization method is implemented to determine the updated set of element properties. If each individual FEM element has only one updating parameter, the updated global mass and stiffness matrices may be generalized as

$$[\mathbf{M}]_{UPD} = \sum_{i=1}^{NE} \sum_{j=1}^{k} P_{ij}\,[\mathbf{m}_{p_j}]_i + [\mathbf{m}_0]_i, \qquad [\mathbf{K}]_{UPD} = \sum_{i=1}^{NE} \sum_{j=1}^{k} P_{ij}\,[\mathbf{k}_{p_j}]_i + [\mathbf{k}_0]_i. \qquad (1, 2)$$

Multiplying equations (1) and (2) with mode shape vector $\{\boldsymbol{\phi}\}$ results in

$$[\mathbf{M}]_{UPD}\,(\phi) = [\mathbf{M}_0]\,(\phi) + [\mathbf{M}_p\,\boldsymbol{\phi}]\,(P), \qquad [\mathbf{K}]_{UPD}\,(\phi) = [\mathbf{K}_0]\,(\phi) + [\mathbf{K}_p\,\boldsymbol{\phi}]\,(P). \quad (3, 4)$$

Two updating processes are developed: (1) selected experimental natural frequencies are used as the updating targets (OPUNF—optimization process for updating natural frequency) and (2) selected mode shapes are the updating targets, with constraints placed as to not alter the natural frequencies (OPUMS—optimization process for updating mode shape).

For ease of implementation, the eigendata differences between the initial FEM and the experimental results are linearly approximated with Jacobian matrices (the sensitivity of eigendata with respect to updating variable vector $\{\mathbf{P}\}$) by equation (5):

$$\{\Delta\boldsymbol{\lambda}\} = [\nabla\boldsymbol{\lambda}]\,(\Delta P), \qquad \{\Delta\boldsymbol{\phi}_i\} = [\nabla\boldsymbol{\phi}_i]\,(\Delta P). \qquad (5)$$

The Jacobian matrix of eigenvalues with respect to the updating variable array is derived following procedures in reference [15]:

$$[\nabla\boldsymbol{\lambda}] = \frac{\partial(\lambda)}{\partial(P)} = \begin{bmatrix} (\phi_1)\,([\mathbf{K}_p\,\phi_1] - \lambda_1\,[\mathbf{M}_p\,\phi_1]) \\ (\phi_1)\,([\mathbf{K}_p\,\phi_2] - \lambda_2\,[\mathbf{M}_p\,\phi_2]) \\ \vdots \\ (\phi_n)\,([\mathbf{K}_p\,\phi_n] - \lambda_n\,[\mathbf{M}_p\,\phi_{n1}]) \end{bmatrix}. \qquad (6)$$

The Jacobian matrix of a specific eigenvector with respect to the updating parameters based on [16] can be formed as

$$[\nabla\boldsymbol{\phi}_i] = \partial(\phi_i)/\partial(P) = [\mathbf{V}_i] + (\phi_i)\{\boldsymbol{\Gamma}_i\}^{\mathrm{T}}. \qquad (7)$$

Using procedures in reference [16] and equation (7), the following relationship can be derived:

$$([\mathbf{K}] - \lambda_i\,[\mathbf{M}])\,[\mathbf{V}_i] = ([\mathbf{K}_p\,\phi_i] - \lambda_i\,[\mathbf{M}_p\,\phi_i]) + [\mathbf{M}]\,(\phi_i)\,(\phi_i)^{\mathrm{T}}([\mathbf{K}_p\,\phi_i] - \lambda_i\,[\mathbf{M}_p\,\phi_i]). \qquad (8)$$

$[\mathbf{V}_i]$ can be resolved with the procedure [15, 16]. Using orthogonality and by further manipulation the coefficient vector $\{\boldsymbol{\Gamma}\}^{\mathrm{T}}$ can be obtained by

$$\{\boldsymbol{\Gamma}\}^{\mathrm{T}} = -(\phi_i)^{\mathrm{T}}[\mathbf{M}]\,[\mathbf{V}_i] - \tfrac{1}{2}\,(\phi_i)^{\mathrm{T}}[\mathbf{M}_p\,\boldsymbol{\phi}_i]. \qquad (9)$$

Equations (5–9) can then be used to form the respective optimization problems to accomplish the updating.

The OPUNF is based on the optimization problem cast as

$$\text{Minimize} \quad \tfrac{1}{2}\{\Delta P\}^{\mathrm{T}}(\Delta P) \qquad \text{subject to} \quad \{\Delta \boldsymbol{\lambda}\} = [\boldsymbol{\nabla\lambda}]\,(\Delta P) \qquad (ILB) \leqslant (\Delta P) \leqslant (IUB). \tag{10}$$

The OPUMS is based on the optimization problem cast as

$$\text{Minimize} \quad \tfrac{1}{2}\sum_{i=1}^{Nm} \|[\mathbf{WT}_i]\,[(\Delta\varphi_i) - [\boldsymbol{\nabla\psi}_i]\,(\Delta P)]\|$$

$$\text{Subject to} \quad \{\mathbf{0}\} = [\boldsymbol{\nabla\lambda}]\,(\Delta P) \qquad (ILB) \leqslant (\Delta P) \leqslant (IUB), \tag{11}$$

where $(\boldsymbol{\Delta\varphi}_i)$ and $[\boldsymbol{\nabla\psi}_i]$ are defined as

$$\{\Delta\boldsymbol{\varphi}_i\} = \frac{(\phi_i^{EXP})}{\sqrt{(\phi_i^{EXP})^{\mathrm{T}}(\phi_i^{EXP})}}\frac{-(\phi_i^{FEM})}{\sqrt{(\phi_i^{FEM})^{\mathrm{T}}(\phi_i^{FEM})}}, \qquad [\boldsymbol{\nabla\psi}_i] = \frac{[\boldsymbol{\nabla\phi}_i^{FEM}]}{\sqrt{(\boldsymbol{\phi}_i^{FEM})^{\mathrm{T}}(\boldsymbol{\phi}_i^{FEM})}}. \quad (12, 13)$$

$[\mathbf{WT}]$ is a weighting matrix, depending on the confidence of the measured experimental data. In this study, it is set as an identity matrix.

Both updating procedures take the form of a linear estimation in a constrained optimization process. If the initial FEM results are close to the actual structure, small updating parameter changes can be expected and the linear estimation works well. However, in practical application, the FEM may not be sufficiently close. In this situation the updating uses a partitioning procedure.

2.4. ELEMENTAL LEVEL UPDATING IMPLEMENTATION

Two updating procedures, NFU (natural frequency updating) and NF-MSU (natural frequency–mode shape updating), are used. The NFU procedure utilizes only the OPUNF process, equation (9), in an iterative manner. The NF-MSU procedure iteratively uses both OPUNF (equation (9)) and OPUMS (equation (10)). The basic concept of the NF-MSU procedure is to update natural frequencies first and then attempt to obtain a better correlation between the updated FEM and experimental mode shapes without changing the natural frequencies. The coding is performed in MATLAB.

A flow diagram illustrating the multiple step updating processes is shown in Figure 1. The overall updating procedure may be outlined as follows:

(1) Obtain the FEA model to be updated and the complementary set of experimental modal data.

(2) Determine the updating parameters, P_i, and optimization variables, ΔP_i, where $P_i = 1 + \Delta P_i$. Assemble the updating coefficient array $\{\mathbf{P}\}$.

(3) Establish allowable upper and lower optimization variables bounds, (PUB^0) and (PLB^0), for the entire procedure: $(PLB^0) \leqslant (\Delta P) \leqslant (PUB^0)$ where $(PLB^0) \leqslant 0$ and $(PUB^0) \geqslant 0$.

(4) Establish allowable upper and lower optimization variables bounds, (IUB^i) and (ILB^i), for a single optimization step: $(ILB^i) \leqslant (\Delta P^i) \leqslant (IUB^i)$ with $(ILB^i) \leqslant 0$ and $\{PUB^i\} \geqslant 0$ and where i represents the ith optimization step.

(5) Apply the natural frequency updating NFU process.

(6) Apply the mode shape updating MSU process.

(7) If necessary, step 5 and step 6 may be repeated to guarantee that the natural frequencies and mode shapes of the updated FEM remain close to the experimental target values.

The updating parameters potentially may lose their practical meaning if the values change drastically in the least squares optimization process. To overcome these potential problems, upper and lower bounds on the updating variables are set. Furthermore, large changes in the updating coefficients may affect the accuracy of the linear estimation via the Jacobian matrix. Therefore, the changes in the optimization variables must be kept small. The optimization variable upper and lower bounds are adjusted after each step [24]. From experience, if the bounds do not exceed 5% of the original value, $(ILB^i) = (-0{\cdot}05)$ and $(IUB^i) = (0{\cdot}05)$, the linear estimation works well. Since each process is formulated as a classical quadratic optimization problem it may be solved using the projection technique [25].

3. SIMULATION STUDY

To evaluate the elemental updating process a computer-based test was performed. Initially, a cantilever beam FEM with varying stiffness and mass is developed and serves as the baseline structure. The varying stiffness and mass values are created by applying a multiplicative constant between 0·8 and 1·2 to the respective elemental matrices. An eigenanalysis of the baseline FEM serves as a simulated experimental modal database. Next, a FEM with a uniform mass and stiffness distribution is developed. The elemental updating procedure is applied to the uniform based FEM in an effort to match the simulated experimental results (varying mass and stiffness distribution). Since the set of multiplicative constants in the baseline FEM (varying mass and stiffness beam) in essence represents the updating coefficient array, a direct comparison to the optimally updated results will allow the capabilities of the updating method to be examined.

The cantilever beam FEM has nine equally spaced consistent mass beam elements with the properties summarized in Table 1. The varying mass and stiffness characteristics are defined by nine multiplicative constants applied to each element's respective mass and stiffness matrices. The vectors are denoted with "Exp" subscript to infer the experimental values while the subscripts "m" and "k" correspond to the respective mass or stiffness element matrix:

$$\{\mathbf{P}_m\}_{Exp} = \{0{\cdot}8,\ 1{\cdot}0,\ 1{\cdot}0,\ 1{\cdot}2,\ 1{\cdot}2,\ 1{\cdot}2,\ 1{\cdot}0,\ 1{\cdot}1,\ 1{\cdot}1\}^{\mathrm{T}},$$

$$\{\mathbf{P}_k\}_{Exp} = \{0{\cdot}8,\ 1{\cdot}0,\ 0{\cdot}9,\ 1{\cdot}0,\ 1{\cdot}0,\ 0{\cdot}9,\ 0{\cdot}9,\ 0{\cdot}9,\ 0{\cdot}9\}^{\mathrm{T}}.$$

These arrays indicate that the mass matrix of element 1 will be multiplied by 0·8 before being placed in the global mass matrix. Similarly, the stiffness for element 3 will be multiplied by a factor of 0·9 before being placed in the global stiffness matrix.

The "experimental" FEM results are created to more closely simulate what would be actually obtained in an experimental modal analysis. In an experiment, the number of modes estimated is usually considerably lower than for a FEM. Therefore, data for only the first five modes will be retained and all higher modes discarded. Since rotational information is difficult to measure, it is rarely included in experimental mode shapes. To retain this realistic characteristic, all rotational degrees of freedom are eliminated from the modal vectors leaving only the translational components. Inherent experimental errors produce a degree of uncertainty in all experimental modal analysis results. To simulate this uncertainty, 2% random noise was added to the natural frequencies and 5% random noise was added to the mode shapes (percentages referenced to the baseline FEM data).

The uniform beam FEM is formed with nine consistent mass beam elements using the geometric and properties as above. In essence this represents the same model as the simulated experimental model (baseline FEM), but with $\{\mathbf{P}_m\}_{FEM} = \{1\}$ and $\{\mathbf{P}_k\}_{FEM} = \{1\}$.

This FEM represents the model that the analyst would initially produce and would be in need of updating before performing any design analysis.

To investigate the capabilities the two different updating procedures, NF-MSU and NFU, are applied. The first approach, NF-MSU, involves both the OPUNF and OPUMS iterative updating algorithms. The FEM is first updated only by OPUNF (based solely natural frequency constraints) and secondly, the OPUMS process is applied to improve the mode shape correlation. The updating procedure attempts to first adjust the natural frequencies then secondly adjust the mode shapes to the experimental results. In NFU, only the OPUNF algorithm is applied to update the FEM. The iterative sequencing of the both NF-MSU and NFU procedure is listed in Table 2.

The data sets discussed above can be summarized as:

(1) ''Baseline Model'' is the FEM with a varying mass and stiffness using the elemental multiplicative constants arrays $\{\mathbf{P}_m\}_{Exp}$, and $\{\mathbf{P}_k\}_{Exp}$.

(2) ''Initial FEM'' is the FEM of the uniform beam.

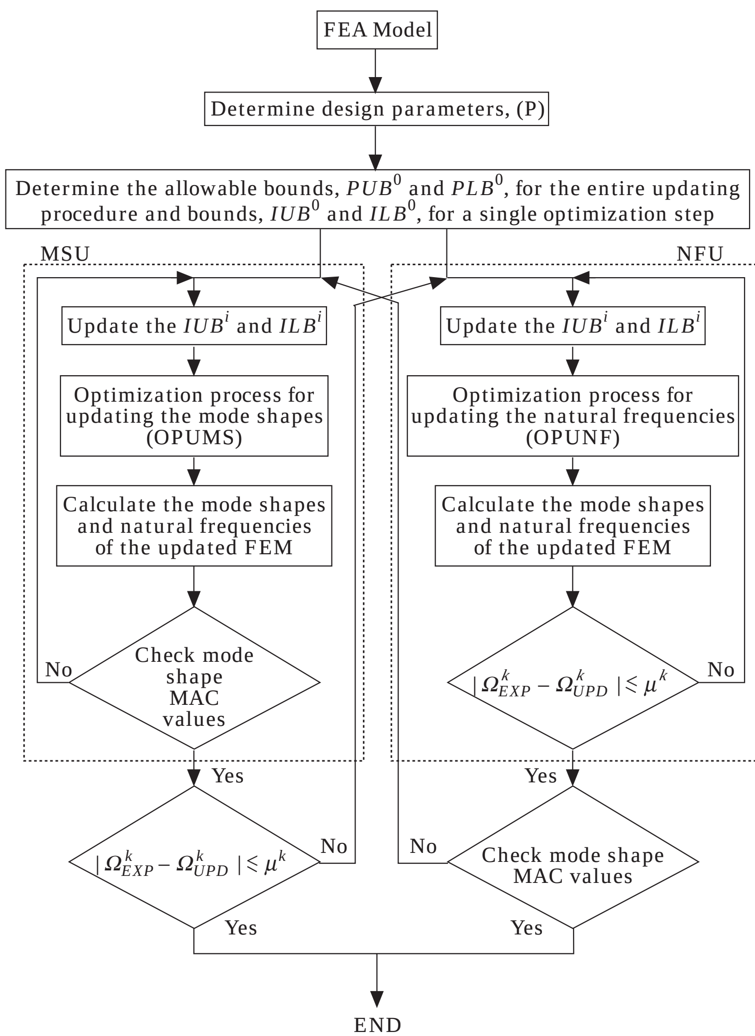

Figure 1. Flow diagram of NF-MSU updating procedure.

TABLE 1

Properties of the uniform cantilever beam

Length (m)	Width (m)	Height (m)	Density (kg/m^3)	Young's modulus (N/m^2)
7·73E − 1	2·413E − 2	6·35E − 3	7·806E + 3	2·07E + 11

(3) "NF-MSU Updated FEM" is the FEM updated by iteratively applying the OPUNF and OPUMS optimization processes.

(4) "NFU Updated FEM" is the FEM updated by using only the OPUNF optimization process.

(5) "Noise Free Experimental Data" is the simulated experimental modal database determined by truncating the FEM in item 1 after mode number five and discarding the rotational degrees of freedom.

(6) "Simulated Experimental Data" is obtained by adding random noise 2% and 5% with reference to the respective natural frequencies and mode shapes from the noise free experimental data described in item 5.

Table 3 compares the natural frequencies of the first five modes from two simulated experimental data sets (with and without noise), the initial FEM, and the updated FEMs. It is observed that both updating procedures, NF-MSU and NFU, produce results where the first five natural frequencies agree very closely to the "experimental" data.

Recall that only five modes are used for the simulated experimental modal databases. For analysis purposes, the updated results for the higher modes also have been included. Figures 2 and 3 compare the differences between the experimental natural frequencies and the three finite element models (initial and two updated FEMs) for the higher truncated modes. Significant improvement in natural frequency correlation is realized even though these higher modes are not explicitly considered in the updating process. It can also be observed that the results from the NF-MSU procedure are better than those obtained from NFU procedure. As would be expected, the updated results are better for the noise free case. However even in the presence of noise, the higher mode natural frequencies for the updated FEM show significant improvement over the original values.

A visual inspection of the modal assurance criteria (MAC) (Table 4) is applied to quantify the mode shape correlation for the first five modes between the various models showed are excellent correspondence with only minor differences. For the experimental data case, the NF-MSU updated FEM produces a slightly better mode shape correlation. For the noise free experimental data case, the NF-MSU updated FEM reveals excellent mode shape correlation with respect to the baseline FEM (MAC values close to unity). Whereas, the NFU updated FEM does not show improvement.

TABLE 2

The iterative sequencing used to update the cantilever beam

	Procedure	Updating process sequence
Simulated experimental data	NF-MSU	OPUNF-OPUNF-OPUMS-OPUMS-OPUMS-OPUNF
	NFU	OPUNF-OPUNF-OPUNF
Simulated noise free experimental data	NF-MSU	OPUNF-OPUNF-OPUMS-OPUMS-OPUMS-OPUMS-OPUNF
	NFU	OPUNF-OPUNF-OPUNF

TABLE 3

Cantilever beam natural frequency (Hz) comparison from the simulated experimental data (with and without noise), the initial FEM and the updated FEMs

		Mode number				
		1	2	3	4	5
	Initial FEM (Hz)	8·8402	55·404	155·184	304·417	504·388
	Simulated experimental data	7·978	49·460	141·138	277·442	462·569
Updating with simulated experimental data	NF-MSU updated FEM	7·978	49·460	141·138	277·442	462·569
	NFU updated FEM	7·978	49·460	141·138	277·442	462·569
	Simulated noise free experimental data	7·974	49·538	141·988	282·010	463·799
Updating with simulated noise free experimental data	NF-MSU Updated FEM	7·974	49·538	141·988	282·010	463·799
	NFU updated FEM	7·974	49·538	141·988	282·010	463·799

A comparison of the updating coefficient array $\{\mathbf{P}\}$ for the four FEMs is shown in Tables 5 and 6. The FEM updated by NF-MSU procedure reveals the best results and is particularly apparent in the noise free experimental data case. The NF-MSU updated coefficients $\{\mathbf{P}\}$ show high accuracy, whereas the NFU procedure does not produce similar improvement. It may be observed that the mass updating coefficients $\{\mathbf{P}_m\}$ show very good correlation with the baseline model for the NF-MSU procedure, even in the presence of noise. However, an interesting observation is that the stiffness updating coefficients $\{\mathbf{P}_k\}$ do not have similar results.

From this simulation, the following observations can be made:

(1) If the experimental data is noise free, the updated results based on the NF-MSU procedure produces excellent correlation with the baseline FEM.

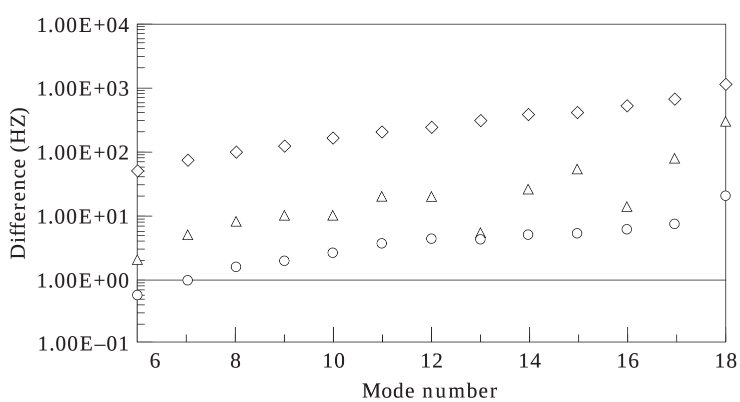

Figure 2. Cantilever beam natural frequency differences for truncated modes (modes 6–18) between the baseline FEM (actual model) and: ◇, initial FEM; ○, NFU updated FEM using the simulated noise free experimental data; △, NF-MSU updated FEM using the simulated noise free experimental data.

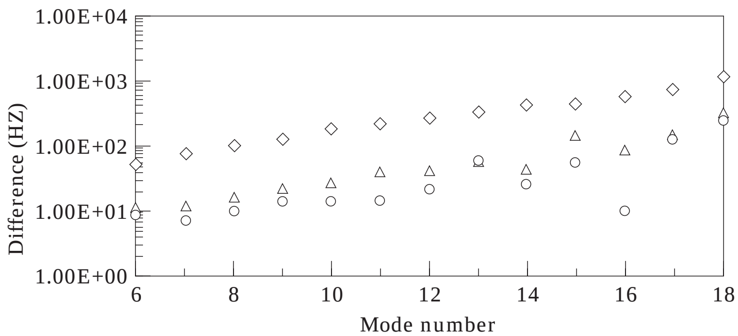

Figure 3. Cantilever beam natural frequency differences for truncated modes (modes 6–18) between the baseline FEM (actual model) and: ◇, initial FEM; ○, NFU updated FEM using the simulated experimental data; △, NF-MSU updated FEM using the simulated experimental data.

(2) The NFU updating coefficients do not converge to the baseline mode. However, the NFU updated results are no worse than the initial FEM.

(3) Both NFU and NF-MSU, produce updated FEMs with excellent correlation to the experimental natural frequencies.

(4) The updating results based on the NF-MSU procedure are better than the NFU procedure. The NF-MSU procedure improves both the frequency and mode shape correlation. However, the mode shape updating process is inherent more computationally intensive and usually costs several times the computation effort than using just the natural frequencies.

(5) Although the NF-MSU procedure is successfully applied in this simulation case, the potential effect of experimental error in mode shape updating remains difficult. It remains a topic to be explored further and its applications have to be carefully judged by the analyst.

TABLE 4

The modal assurance criteria (MAC) for the cantilever beam between the baseline FEM (actual model), initial FEM, simulated experimental data, NFU updated FEM, and NF-MSU updated FEM

	MAC(1,1)	MAC(2,2)	MAC(3,3)	MAC(4,4)	MAC(5,5)
Baseline FEM/initial FEM	0·99996	0·99859	0·99977	0·99705	0·99505
Baseline FEM/simulated experimental data	0·99901	0·99957	0·99898	0·99974	0·99936
Baseline FEM/NFU updated FEM using noise free experimental data	0·99996	0·99829	0·99951	0·99688	0·99996
Baseline FEM/NF-MSU updated FEM using noise free experimental data	1·00000	1·00000	1·00000	1·00000	1·00000
Baseline FEM/NFU updated FEM using simulated experimental data	0·99992	0·99926	0·99987	0·99705	0·99516
Baseline FEM/NF-MSU updated FEM using simulated experimental data	0·99999	0·99994	0·99976	0·99986	0·99996

4. DESIGN CHANGE EVALUATION OF A PORTAL FRAME

A portal frame (Figure 4) constructed by joining three separate aluminum beams via bolted connections is the test object. The approximate dimensions of the two vertical side members are $508 \times 25{\cdot}4 \times 6{\cdot}35$ mm and with the cross member beam $406{\cdot}4 \times 25{\cdot}4 \times 6{\cdot}35$ mm. The joints were bolted connections made with aluminum "L" brackets having approximate dimensions of $19{\cdot}05 \times 25{\cdot}4 \times 3{\cdot}18$ mm on each side.

The analysis evaluates the new modal characteristics if an additional structural member is placed between the middle of vertical sides of the portal frame as shown in Figure 5 (dotted member). The added member was also bolted and has the same dimensions as the top cross member. The general evaluation procedure can be outlined as follows:

(1) An experimental modal analysis is performed on the portal frame hardware.

(2) A FEM of the portal frame is developed.

(3) The FEM is updated with the modal test data.

(4) The updated FEM is modified to include the new center structural member.

(5) An eigenanalysis is performed to predict the new structural dynamic characteristics.

(6) The change is physically made and another experimental modal analysis performed.

(7) The FEM results are compared to the experimental modal data to evaluate the capabilities to predict the changes in the structural dynamic behavior.

The experimental modal analysis used 33 grid points as illustrated in Figure 5 with a reference accelerometer mounted at node 8. Standard impact testing was used and the modal data for the first six modes were estimated with StarModal [24].

The FEM was developed with frame elements and 33 nodal points which matched the geometric locations of the experimental test points shown in Figure 5. All the joints were modelled by equating the respective degrees of freedom of the connected members.

Both updating procedures, NFU and NF-MSU, were applied to update the portal frame FEM with the experimental data from the first five modes. The section area, A, and the moment of inertia, I, for each frame element were used, yielding a total of 64 updating parameters. The iterative sequence for the updating procedures are listed in Table 7. The natural frequency updating was implemented with a partitioning process to ensure success with the linear estimation. Figures 6 and 7 show the updating parameter bounds and the corresponding updating results. Wider bounds are set around the boundaries and joints because initially less modelling confidence exists around these regions. Several trends can be observed from the updating parameter results:

(1) Both the updating coefficient array (P_A and P_I) possess symmetric distributions around elements sixteen and seventeen. This is to be expected due to the inherent symmetry of the portal frame.

(2) The moment of inertia updating coefficients surrounding the fixed–fixed boundaries (at node 1 and node 33) decrease more than the other elements. This is reasonable since more flexibility would be expected than the modelled fixed–fixed conditions.

(3) The largest changes to the moment of inertia updating coefficients occur around elements 10–13 and elements 20–23. This corresponds to the joint regions and reflects that less modelling confidence exists. This indicates more attention to the joint modelling is necessary.

A comparison of the respective natural frequencies is shown in Table 8. The initial FEM differs from the experimental values by approximately 10–20%. After updating, both effective FEMs match the experimental frequencies for the first five modes. Furthermore, both reveal a good correlation for the sixth natural frequency (less than 4%) even though it was not explicitly used in the updating. The MAC values are listed in Table 9. A visual inspection of the mode shapes and the MAC values indicate good correlation. From

TABLE 5

A comparison of the mass matrix updating coefficient $\{\mathbf{P}_m\}$, *from the baseline FEM, the initial FEM, and the respective NFU and NF-MSU updated FEMs*

		Element number								
		1	2	3	4	5	6	7	8	9
	Initial FEM	1	1	1	1	1	1	1	1	1
	Baseline Model	0·8	1	1	1·2	1·2	1·2	1·2	1·1	1·1
Model updated with simulated noise free experimental data	NFU Procedure	1·0063	1·0392	1·0681	1·1273	1·0910	1·1059	1·0432	1·0510	1·1650
	NF-MSU Procedure	0·8197	1·0025	1·0134	1·2079	1·2145	1·2094	1·0091	1·1086	1·1091
Model updated with simulated experimental data	NFU Procedure	1·0083	1·0730	1·0936	1·1057	1·1244	1·0861	1·0613	1·0571	1·1538
	NF-MSU Procedure	0·8645	1·0256	1·0118	1·2416	1·2024	1·2268	1·0463	1·0975	1·1010

TABLE 6

A comparison of the stiffness matrix updating coefficient $\{\mathbf{P}_k\}$, *from the baseline FEM, the initial FEM, and the respective NFU and NF-MSU updated FEMs*

		Element number								
		1	2	3	4	5	6	7	8	9
	Initial Model	1	1	1	1	1	1	1	1	1
	Baseline Model (Actual Model)	0·8	1	0·9	1	1	0·9	0·9	0·9	0·9
Model updated with simulated noise free experimental data	NFU Procedure	0·8397	0·9492	0·9568	0·8959	0·9107	0·8757	0·9326	0·9606	0·9938
	NF-MSU Procedure	0·8087	1·0035	0·9089	1·0059	1·0108	0·9070	0·9080	0·9088	0·9025
Model updated with simulated Experimental data	NFU Procedure	0·8504	0·9431	0·9392	0·9150	0·8787	0·8963	0·9079	0·9272	0·9918
	NF-MSU Procedure	0·8268	1·0557	0·8259	1·0279	0·9223	0·9704	0·9441	0·8492	0·9634

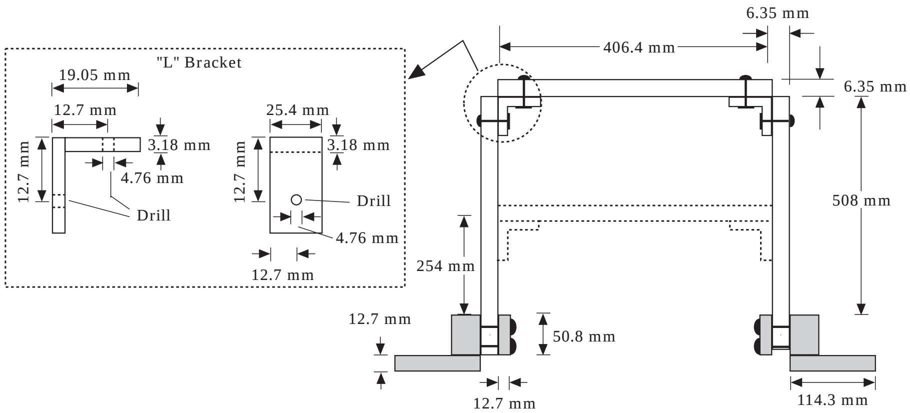

Figure 4. Portal frame structure details. The modification is indicated by the center member shown by the dotted line (. . . . . .).

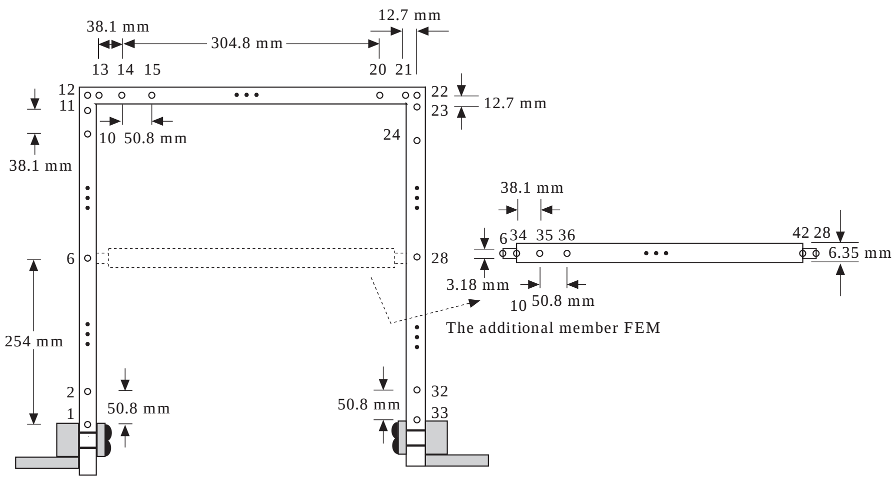

Figure 5. The FEM nodal points of the portal frame.

Tables 8 and 9, it can be observed that the updated effective FEMs are closer to the experimental modal characteristics than the initial FEM. Furthermore, the NF-MSU updated effective FEM reveals slightly better mode shape correlation with the experimental results.

Table 7

The iterative sequencing of both the NFU and NF-MSU procedures for updating the portal frame FEM. Note, the bold **OPUNF** *indicates that three even iterative partitions were used to update the natural frequencies*

	Updating process sequence
NFU procedure	**OPUNF**-OPUNF
NF-MSU procedure	**OPUNF**-OPUNF-OPUMS-OPUMS-OPUMS-OPUNF

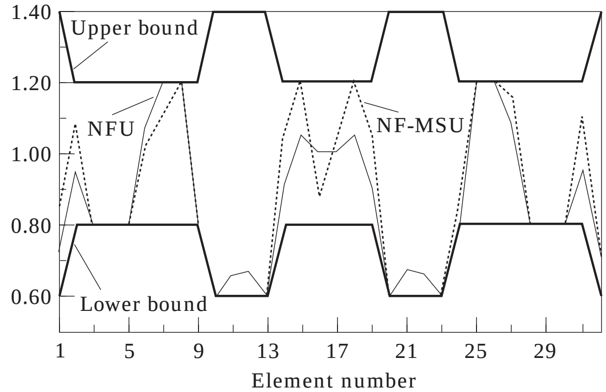

Figure 6. Moment of inertia updating coefficients, P_A, for the portal frame FEM obtained via the respective NFU and NF-MSU updating procedures.

Next, the effect of adding of the center cross member was modelled and evaluated. Three FEMs for the modified portal frame were developed based on (1) the original FEM, (2) the NFU effective FEM, and (3) the NF-MSU effective FEM. All modified portal frame FEMs have a total of 42 frame elements with 42 nodes, using the same co-ordinates as the test grid points illustrated in Figure 5. In all the FEMs, the effect of bolted joints for added cross member is considered. The connection is modelled as a welded joint, however the "L" bracket member thickness is decreased as shown in Figure 5. An eigenanalysis was performed with each FEM. The structural change was physically made by adding the center cross member between the two vertical sides of the portal frame. A modal test was then performed with 42 test points.

The natural frequencies for the modified portal frame from the experiment and predictions are listed in Table 10. The original FEM produced natural frequency differences (with respect to the experimental results) around 10% and 18% in the first and second modes. Whereas, the natural frequency differences from the modified effective FEMs are within 5% for the first five modes. Furthermore, both the modified effective FEMs also reveal excellent natural frequency prediction (differences within 4·1%) for the sixth mode which was not included explicitly in the updating. It is obvious that the natural

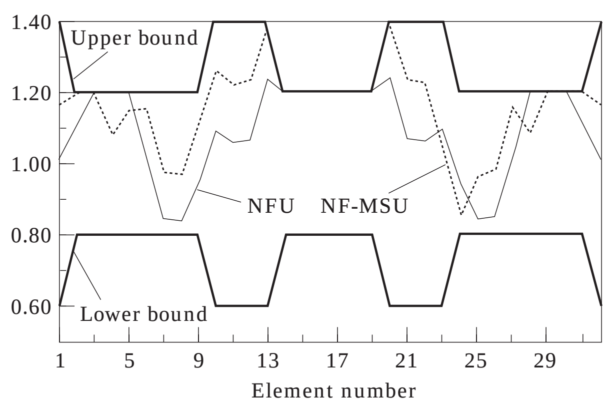

Figure 7. Area updating coefficients, P_A, for the portal frame FEM obtained via the respective NFU and NF-MSU updating procedures.

TABLE 8

Portal frame natural frequency (Hz) comparison from the experiment, initial FEM, and updated FEMs

	Modal number					
	1	2	3	4	5	6
Experiment (Hz)	16·89	83·20	109·06	131·98	287·50	290·95
Initial FEM	20·21	89·14	125·71	159·39	315·09	330·11
Difference (%)	19·67	7·14	15·27	20·77	8·99	13·46
Effective FEM obtained via NFU	16·89	83·20	109·06	131·98	287·50	302·44
Difference (%)	0	0	0	0	0	3·95
Effective FEM obtained via NF-MSU	16·89	83·20	109·06	131·98	287·50	296·61
Difference (%)	0	0	0	0	0	1·95

frequency predictions based on either of the updated effective FEMs are more accurate than the original (non-updated) FEM.

The MAC between experiment and the respective FEMs are listed in Table 11. The modified original FEM reveals poor correlation for modes three (MAC = 0·54) and four (MAC = 0·1). Both the modified effective FEMs reveal excellent correlation for the first six modes, with the exception of mode four. The mode shape correlation for the third mode has shown significant improvement from a MAC value 0·54 from the original FEM to 0·98 for both the effective FEMs. The low correlation for mode four in all the FEMs can be attributed to a poor estimation of the experimental mode shape. Inspection of the experimental data showed that accelerometer reference location was near a node in mode four, resulting in unreliable data. Any conclusions drawn about mode four must be treated carefully.

The following conclusions can be made from these results:

(1) The FEM updating is performed successfully and both the effective FEMs have excellent correlation with the experimental modal data. Overall, the NF-MSU procedure reveals slightly better results.

(2) Both the effective FEMs show better dynamic predictions for the modified portal frame than the original FEM. Again, the new dynamic characteristics predicted based on the NF-MSU updated effective FEM are better than the other FEMs.

TABLE 9

The modal assurance criteria (MAC) for the portal frame between the experiment, initial FEM, NFU updated effective FEM, and NF-MSU updated effective FEM

	MAC (1,1)	MAC (2,2)	MAC (3,3)	MAC (4,4)	MAC (5,5)	MAC (6,6)
Experiment/initial FEM	0·9984	0·9972	0·9936	0·9908	0·9673	0·9948
Experiment/NFU effective FEM	0·9984	0·9980	0·9952	0·9874	0·9339	0·9193
Experiment/NF-MSU effective FEM	0·9984	0·9986	0·9974	0·9922	0·9805	0·9708

TABLE 10

Modified portal frame natural frequency (Hz) comparison from the experiment, modified initial FEM, the modified NFU updated effective FEM, and the modified NF-MSU updated effective FEM

	Modal number					
	1	2	3	4	5	6
Experiment (Hz)	22·09	80·84	116·91	138·56	294·13	310·75
Initial FEM	24·19	95·32	126·09	140·80	317·7	346·75
Difference (%)	9·51	17·91	7·85	1·62	8·01	11·58
Predictions based on the Effective NFU FEM	21·07	83·77	114·72	131·37	294·70	323·29
Difference (%)	4·8	3·62	1·87	5·19	0·19	4·04
Predictions based on the Effective NF-MSU FEM	21·03	84·02	114·86	131·28	294·73	318·16
Difference (%)	4·8	3·93	1·75	5·25	0·2	2·38

TABLE 11

The modal assurance criteria (MAC) for the modified portal frame between the experiment, modified initial FEM, the modified NFU updated effective FEM, and the modified NF-MSU updated effective FEM

	MAC (1,1)	MAC (2,2)	MAC (3,3)	MAC (4,4)	MAC (5,5)	MAC (6,6)
Experiment/predictions based on the original (initial) FEM	0·9946	0·9837	0·5392	0·1042	0·9655	0·8482
Experiment/predictions based on the NFU effective FEM	0·9948	0·9914	0·9835	0·5919	0·9645	0·7604
Experiment/predictions based on the NF-MSU effective FEM	0·9948	0·9900	0·9847	0·5921	0·9826	0·8265

5. SUMMARY

This work has presented a method that allows an effective FEM to be directly developed from an experimental modal database. The computational model geometry is based on the experimental test grid and connectivity and uses beam and/or frame elements. An iterative updating process at the element level forms the effective FEM. The simplified nature of the FEM and the updating process produces a model capable of describing the low order global dynamic characteristics. Furthermore, the effective model can be readily changed to examine the effects of potential design changes. The approach sacrifices a certain level of detail for the sake of rapid development time.

Both the proposed updating procedures, NFU and NF-MSU, have shown to be flexible. If the natural frequencies and mode shapes are updated simultaneously, theoretically, the implementation is not difficult. However, due to experimental errors, at least two hurdles may be encountered: (1) it is difficult to evaluate the relationships between the natural frequency and mode shape errors, (2) the updated results might be sensitive to the experimental errors in a specific mode shape degree of freedom (DOF).

Although the present development concentrated on beam/frame elements, the proposed procedures are applicable to any general element type and structure. The updating can be accomplished without a complete mode shape degree of freedom description. A high level of flexibility exists in selecting the number of modes and the respective DOFs to be used in the updating. In other words, an incomplete modal database is suitable for updating. Also, the updating parameters can easily be selected to best meet the demands of the problem. A simple multiplicative constant or a more specific parameter such as the individual element area moment of inertia (I) may be used. Another important advantage is that the mode shape updating is accomplished without any model condensation or expansion.

The analysis of the computational study showed that both the proposed updating procedures, NFU and NF-MSU, are accurate and flexible:

(1) For the simulated cantilever beam the updated FEMs show excellent natural frequency and mode shape correlation in the target modes. The method is capable of producing a FEM that can accurately replicate global low order experimental dynamic characteristics. Natural frequencies typically were within 0·01% and MAC values greater than 0·99 for a simulated noise free case. When experimental modal analysis uncertainty was simulated, similar results were obtained (natural frequencies within 0·1% and MAC values greater than 0·98).

(2) Improved correlation in the higher modes, not explicitly included in the updating was observed.

(3) It is observed that the updated results based on the NF-MSU procedure are slightly better than NFU procedure. The NF-MSU updating procedure improves both the natural frequency and mode shape correlation whereas NFU updating procedure only improves the natural frequency correlation. However, there is no evidence that the NFU updated model degrades the mode shape correlation in relation to the initial FEM.

(4) The NFU procedure has the advantages of: (1) less required experimental effort (only one test point is required to determine the target natural frequencies), and (2) less computing effort.

The application of the procedure to produce an effective FEM to actual structural redesign scenario was successful. The developed effective FEM produced superior results in relation to the original FEM. Observations for the study can be summarized as:

(1) An effective FEM suitable for redesign analysis can be produced with actual experimental modal and its inherent uncertainty. The effective FEM of the portal frame produced natural frequencies within 5% and MAC values typically above 0·95. The results showed significant improvement over the initial FEM where natural frequencies differences greater than 15% were observed.

(2) The redesign analysis of the portal frame showed that the effective FEM produced considerably more accurate results than the original model with natural frequency predictions within 5·0% and MAC values above 0·98, with the exception of one mode. This is a significant result in light that the evaluated design change was rather radical and not a minor perturbation.

The study has demonstrated that the updated FEM retains consistency and physical significance such that engineering design analysis is feasible. Since, the prediction of the new dynamic characteristics is directly based on the effective FEM the updating is very critical and requires careful consideration by the analyst. The present results indicate that if handled properly the method is capable of producing accurate design change predictions in a rapid fashion. However, it is realized that the analysis performed to date is rather simple in nature. Further work is required to examine the method's capabilities to handle more realistic mechanical systems and possible design changes.

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APPENDIX: NOMENCLATURE

$[\mathbf{M}_0]/\mathbf{K}_0]$	mass/stiffness matrix of the initial FEM whose entries are independent of the updating parameters
$[\mathbf{M}_0]/\mathbf{K}_p]$	mass/stiffness matrix of the initial FEM whose non-zero entries are dependent on the updating parameters

$[\mathbf{M}]/[\mathbf{K}]$	Global mass/stiffness matrix of the initial FEM ($[\mathbf{M}] = [\mathbf{M}_0] + [\mathbf{M}_p]$; $[\mathbf{K}] = [\mathbf{K}_0] + [\mathbf{K}_p]$)
$[\mathrm{M}]_{\mathrm{UPD}}/[\mathrm{K}]_{\mathrm{UPD}}$	updated (modified) global mass/stiffness matrix
$[\mathbf{m}_0]_i/[\mathbf{k}_0]_i$	mass/stiffness submatrix of the FEM ith element whose entries are independent of the updating parameters
$[\mathbf{m}_{p_j}]_i/[\mathbf{k}_{p_j}]_i$	mass/stiffness submatrix of the FEM ith element whose non-zero entries involve components having the jth type of updating parameter
$[\mathbf{m}_p]_i/[\mathbf{k}_p]_i$	mass/stiffness submatrix of the FEM ith element whose non-zero entries are dependent on the updating parameters ($[\mathbf{m}_p]_i = \Sigma[\mathbf{m}_{p_j}]_i$: $[\mathbf{k}_p]_i = \Sigma[\mathbf{k}_{p_j}]_i$)
$[\mathbf{m}]_i/[\mathbf{k}]_i$	mass/stiffness submatrix of the FEM ith element in global co-ordinates ($[\mathbf{m}]_i = \Sigma[\mathbf{m}_{p_j}]_i + [\mathbf{m}_0]_i$; $[\mathbf{k}]_i = \Sigma[\mathbf{k}_{p_j}]_i + [k_0]_i$)
$(\phi_i)_{FEM}$	FEM mode shape for the ith mode
$\{\phi_i\}_{EXP}$	Experimentally measured mode shapes for the ith mode
λ_i	Eigenvalue for the ith mode
Ω_i	natural frequency for the ith mode; $\lambda_i = \Omega_i^2$
$\{\Delta\boldsymbol{\lambda}\}$	Difference vector between the FEM and experiment for specified modes
$[\nabla\lambda]$	Jacobian matrix of the eigenvalues with respect to the updating parameters
$[\nabla\boldsymbol{\phi}_i]$	Jacobian matrix of the mode shape for the ith mode with respect to updating parameters
P_{ij}	the jth type of updating parameter corresponding to the ith element of a FEM
$\{P\}$	an updating coefficient array which represents all updating parameters
$(\boldsymbol{\Delta}P)$	the changes of the updating coefficient array ($\{\Delta P\} = \{P\} - \{1\}$)
$[\mathbf{WT}]$	weighting matrix
$\Vert\ \ \Vert$	Euclidean norm
$(IUB)/(ILB)$	allowable upper/lower change bounds of the optimization variables
$\{\ \ \}^{\mathrm{T}}$	transpose of a matrix or vector
N_mN	Number of the measured modes
NE	number of FEM elements
OPUNF	Optimization Process for Updating Natural Frequencies
OPUMS	Optimization Process for Updating Mode Shapes
NFU	Natural Frequency Updating procedure
NF-MSU	Natural Frequency and Mode Shape Updating procedure